SLAC-PUB-95-6965
August 1995

Transverse Multibunch Modes for Non-Rigid Bunches, Including Mode Coupling *

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Abstract

A method for computing transverse multibunch growth rates and frequency shifts in rings, which has been described previously [1, 2], is applied to the PEP-II B factory. The method allows multibunch modes with different internal-bunch oscillation modes to couple to one another, similar to single-bunch mode coupling. Including coupling between the multibunch modes gives effects similar to those seen in single-bunch mode coupling. These effects occur at currents that are lower than the single-bunch mode coupling threshold.

1 Physical Motivation

Instability due to transverse mode coupling cannot occur unless two requirements are met. First, there must be a mechanism for the rigid ($m = 0$) motion to drive the head-tail ($m = 1$) motion, or vice-versa (only considering coupling between these two modes). In the case of a single bunch, this driving comes about because the head of the bunch sees no wakefield, whereas the tail of the bunch sees the wakefield of the entire bunch. The second requirement is that the frequencies of the two types of motion must be similar so that one mode can resonantly drive the other. In the case of a single bunch, this comes about because the average transverse wake in the bunch usually acts as an effective defocussing force on the bunch centroid, reducing the oscillation frequency of the $m = 0$ mode to the point where it eventually equals the frequency of one of the $m = 1$ modes.

Now consider multibunch modes. A transverse multibunch mode is a mode where each bunch in the train executes identical types of oscillations: for example, rigid oscillations ($m = 0$), or head-tail oscillations ($m = 1$). Calculations up to this point have typically treated these multibunch modes as uncoupled. This paper shows that important effects are missed when coupling between these modes is ignored.

One expects some coupling between the multibunch modes for the reasons outlined in the first paragraph. Consider an $m = 0$ multibunch oscillation. Such an oscillation will induce a wakefield, which in general has a nonzero slope in most places. This nonzero slope means that each bunch sees a different wakefield at the head and the tail. Thus, an $m = 0$ multibunch oscillation can drive an $m = 1$ multibunch oscillation. If the current is high enough and/or the bunches are close enough together so that the wakefields extend from one bunch to the next, the difference in wake seen across one bunch due to previous bunches can be significant, even compared to the difference in wake seen across the bunch due to its own wakefield. This can occur even when the wavelength of the wakefield in question is much longer than the length of the bunch. B factories such as PEP-II at SLAC [3] operate at high currents with a large number of bunches, and thus one might expect this driving to be significant.

A broadband impedance corresponds to a wakefield that is short range; the wakefields do not typically extend from one bunch to the next. Therefore, when only a broadband impedance exists, mode coupling is adequately described by looking at a single bunch. But for narrow-band impedances, such as cavity higher order modes, which correspond to wakefields that extend over long distances, a bunch can create wakefields that are visible to several bunches behind it. Thus, these narrow-band impedances can easily be the mechanism through which the $m = 0$ and $m = 1$ multibunch modes drive one another. The decay time for the cavity higher order modes in the PEP-II B factory is much longer than the time between bunches [3, 4], and thus this driving can be significant.

*Work supported by Department of Energy contract DE–AC03–76SF00515.

Presented at the International Workshop on Collective Effects and Impedance for B-Factories, Tsukuba, Japan, 12–17 June, 1995.

Since these narrow-band impedances couple the various bunches together, they also may cause frequency shifts and growth rates in the multibunch modes that are comparable to the synchrotron frequency. Thus, there are multibunch modes whose frequencies are shifted in such a way that the corresponding $m = 0$ and $m = 1$ multibunch mode frequencies coincide at currents that are smaller than the current at which the two modes coincided if only a single bunch was considered. Multibunch mode coupling is therefore expected to give sharp increases in growth rates at currents that are lower than the corresponding current at which single-bunch mode coupling occurs.

2 Basic Formalism

Stability of the beam is determined as follows:

- Start with equally spaced bunches, each bunch having an identical Gaussian bunch distribution.
- Consider small perturbations about that distribution; use the Vlasov equation to obtain an eigenvalue equation for the oscillation frequency of these perturbations.
- If that oscillation eigenvalue has a positive imaginary part, the beam is unstable.

After much manipulation, the resulting eigenvalue equation becomes [1]

$$\phi_m(\Omega) = \sum_{n=0}^{\infty} K_{m+n}(\Omega + p\omega_0)F_n(\Omega)\phi_n(\Omega) \tag{1}$$

$$F_n(\Omega) = \frac{1}{2^n n!}\sum_{k=0}^{n}\binom{n}{k}\frac{[\omega_y - (n-2k)\omega_z]^2}{\Omega^2 - [\omega_y - (n-2k)\omega_z]^2} \tag{2}$$

$$K_k(\omega) = -i\frac{r_0 c^2 \beta_y N M}{\gamma_0 L^2 \omega_y}\sum_{\alpha}\left(\frac{\sigma_\ell}{\beta_0 c}\right)^k (\omega + M\alpha\omega_0)^k Z_\perp(\omega + M\alpha\omega_0)e^{-\sigma_\ell^2(\omega+M\alpha\omega_0)^2/\beta_0^2c^2}, \tag{3}$$

where ω_0 is the angular revolution frequency of the ring, ω_y is the betatron frequency, ω_z is the synchrotron frequency, r_0 is the classical radius of the electron (or the corresponding value for whatever type of particle the beam consists of), c is the speed of light, β_y is the average β function, N is the number of particles in a bunch, M is the number of bunches, γ_0 is the nominal beam energy divided by the rest-mass energy of the particle, L is the length around the ring, σ_ℓ is the bunch length, $\beta_0 c$ is the nominal particle velocity, and $Z_\perp$ is the transverse impedance. The coherent frequency in the bunch frame is Ω, and it will have a positive imaginary part if the beam is unstable. p is an integer index describing the multibunch mode number; it can take on the values $0 \ldots M-1$. A feedback system is modelled by adding an additional term to K_k with $Z_\perp(q\omega_0 + \Omega)$ replaced by $Z_{\mathrm{FB}}(q\omega_0 + \Omega)e^{-2\pi i q\Delta s/L}$, where Z_{FB} is the Fourier transform of the feedback response, and Δs is the distance between the pickup and kicker. Here q is the combination $p + M\alpha$ in equations (1–3). See [1] for more details.

3 Impedance Model Used for PEP-II

A computer program was written that computes the multibunch mode eigenfrequencies as described in [1, 2]; the program is able to use an arbitrary impedance. The transverse impedance used is a sum of several terms, each corresponding to a different source of impedance. Terms for the resistive wall, an inductive part, high-frequency tails for the cavities, and cavity higher order modes are used.

3.1 Resistive Wall

The resistive-wall impedance can be taken directly from [4]. It is given by the formula

$$Z_\perp^{\mathrm{RW}}(\omega) = -i\sqrt{2}\frac{R_{\mathrm{RW}}}{\sqrt{-i\omega/\omega_0}}, \tag{4}$$

where ω_0 is the angular revolution frequency. R_{RW} is 1.175 MΩ/m horizontally, and 1.61 MΩ/m vertically.

3.2 Inductive

Many devices, such as bellows, BPM's, and slots, give an impedance that is primarily inductive. The inductive part is obtained by scaling the longitudinal inductive impedance of 83.3 nH [4] by $2c/\omega b^2$ [5], where b is a characteristic size of the beam pipe. Worst-case values are obtained by performing this scaling with $b = 2.5$ cm (the vertical size of the beam pipe in the bends [3]).

The impedance will not be constant for all frequencies; it is expected to begin to roll off at high frequencies. Since the average behavior at high frequencies is expected to be similar to that of a cavity, a high frequency roll-off of $\omega^{-3/2}$ is used [6]. On average, the roll-off is estimated to be around 10 GHz [6]. Thus, the model

$$Z_{\perp}^{\mathrm{Ind}}(\omega) = \frac{-iL}{(1 - i\omega/\omega_C)^{3/2}} \tag{5}$$

is used, with $L = 83.3$ nH, and $\omega_C = 10$ GHz.

An improvement on this model would be to consider the loss factor from these "inductive" elements as well. The choice of the cutoff also needs further study [7].

3.3 Cavity Tails

It is well known that the longitudinal impedance of a single cavity rolls off at high frequency as $\omega^{-1/2}$ [8]. A simple model with the appropriate high-frequency roll-off is

$$Z_{\parallel}^{\mathrm{Tail}}(\omega) = iA\left[\left(1 + \frac{\omega}{\omega_0 + i\alpha}\right)^{-1/2} - \left(1 - \frac{\omega}{\omega_0 - i\alpha}\right)^{-1/2}\right]. \tag{6}$$

This model is fit to a model of the cavity run through ABCI for $m = 1$ [4, 9] with the known higher order modes removed. The parameters are found to be $A = 45.1344$ kΩ/m, $\omega_0 = 2.4$ GHz, and $\alpha = 1.34722$ GHz [7]. This model for the longitudinal impedance is then turned into a transverse impedance using the Panofsky-Wentzel theorem [5].

Since the cavities are localized, the impedance must be multiplied by the ratio of the average β function at the cavities to the average β function used in equation (3) (typically the average β function of the ring) [1, 10, 11, 12]. For the PEP-II LER, these values are 12.0066 m and 18.5074 m respectively in the vertical direction [13].

3.4 Cavity Higher Order Modes

The transverse cavity higher order modes can be obtained directly from [4]. Each mode is considered to be a single resonator of the form [5]

$$Z_{\perp}^{\mathrm{Res}}(x\omega_R) = \frac{Z_{\mathrm{eff}}}{x + iQ(1 - x^2)}. \tag{7}$$

As for the cavity tails, the impedance must be multiplied by the ratio of the average β function at the cavities to the average β function used in the formulas.

3.5 Other Sources Not Included

Potentially large resonances due to beam position monitors and the interaction region chamber have not been included in this calculation. A preliminary estimate indicates that these resonances will probably have only a small effect, but enough that they should be included in the calculation.

4 Results for PEP-II

The impedance model from section 3 was used to compute multibunch modes as described in [1, 2]. The computations shown here are for the vertical direction in the low-energy ring, which typically gives the worst case results. Table 1 gives the relevant parameters. The operating current assumed in this calculation is 3.159 A, not 3 A, because the higher current is the total beam current that gets the single-bunch current

betatron tune	ν_y, ν_β	37.64
synchrotron tune	ν_s	.03362246
bunch length	σ_ℓ	1 cm
circumference	L	2199.318 m
average β function	$\langle\beta_y\rangle$	18.5074 m
average β function at cavities	$\langle\beta_y\rangle_{\mathrm{CAV}}$	12.0066 m
energy	E	2.5 GeV
operating current	I	3 A
number of cavities	N_{CAV}	24
r.f. frequency	f_{RF}	476 MHz
harmonic number	h	3492
number of bunches	k_B	1658
number of bunch buckets	M	1746
vertical damping time	τ_y	.0576 s

TABLE 1: Parameters for the PEP-II B factory low-energy ring that are used in the calculations here [3, 13]. Note that energy, current, and number of cavities are worst-case values.

right. This current gives the worst-case values for growth rates for multibunch modes when coupling is ignored [14]. Also, getting the single-bunch current right gives the correct results for single-bunch mode coupling. The combination of these two effects would cause one to expect that getting the single-bunch current right will give the worst-case growth rates for multibunch mode coupling as well.

Only the $m = 0$ and $m = 1$ modes are computed in this calculation. Since many of the impedances used have potentially significant contributions at high frequencies, it would be useful to also include terms for higher m (see equation (3) and [15, 16, 17]).

First, for the purposes of comparison, one can examine the results for single-bunch mode coupling, shown in Fig. 1. The single-bunch mode coupling threshold is approximately 14.5 A (8.3 mA per bunch). The behavior of single-bunch mode coupling will determine the average behavior for multibunch mode coupling.

Next, one can compute the frequencies of the multibunch modes. Frequencies for multibunch modes when coupling is not considered are shown in Fig. 2. Narrow-band impedances cause the various multibunch modes to have different frequency shifts; these frequency shifts are approximately centered about the frequency shift due to broadband impedances only, which is what would be seen for only a single bunch. The modes with the largest downward shifts can have the frequencies of their corresponding $m = 0$ and $m = 1$ modes coincide as low as 10 A in this case, much lower than where single-bunch mode coupling occurred. Since there is no coupling between the $m = 0$ and $m = 1$ modes, the frequencies shift almost exactly linearly with current.

If coupling between the $m = 0$ and $m = 1$ multibunch modes is included, the picture of the frequency shifts appears very similar (Fig. 3). The mode frequencies now shift nonlinearly with current, and the frequencies for many modes coincide at even lower currents than if coupling is ignored.

Now consider the growth rates of the multibunch modes. The $m = 0$ modes without mode coupling are shown in Fig. 4. The growth rates increase linearly with current, and are thus nonzero even for small currents. The largest growth rates are significantly larger than growth rates that result from single-bunch mode coupling. Fig. 5 shows the $m = 0$ modes with coupling. Now the growth rates no longer increase linearly with current. Growth rates increase sharply near the single-bunch mode coupling threshold for modes that had low growth rates when coupling wasn't considered. These are modes which don't see any of the narrow-band resonances, and thus involve little bunch-to-bunch coupling; their behavior therefore imitates single-bunch mode coupling. Modes that had high growth rates when mode coupling wasn't considered are affected only slightly by mode coupling because their growth rates were much larger than the characteristic growth rates from mode coupling (see Fig. 1).

The $m = 1$ modes without coupling are shown in Fig. 6. Since coupling is ignored, the growth rates increase linearly with current. The growth rates are much smaller than growth rates that occur once single-bunch mode coupling occurs. Thus, when coupling of the $m = 1$ modes with the $m = 0$ modes is considered, a significant increase in growth rates due to multibunch mode coupling is found, as shown in Fig. 7. Growth rates start to increase sharply at currents close to where the frequencies of the $m = 0$ and $m = 1$ multibunch modes coincided (see Fig. 3). This current is significantly lower than the threshold current for single-bunch

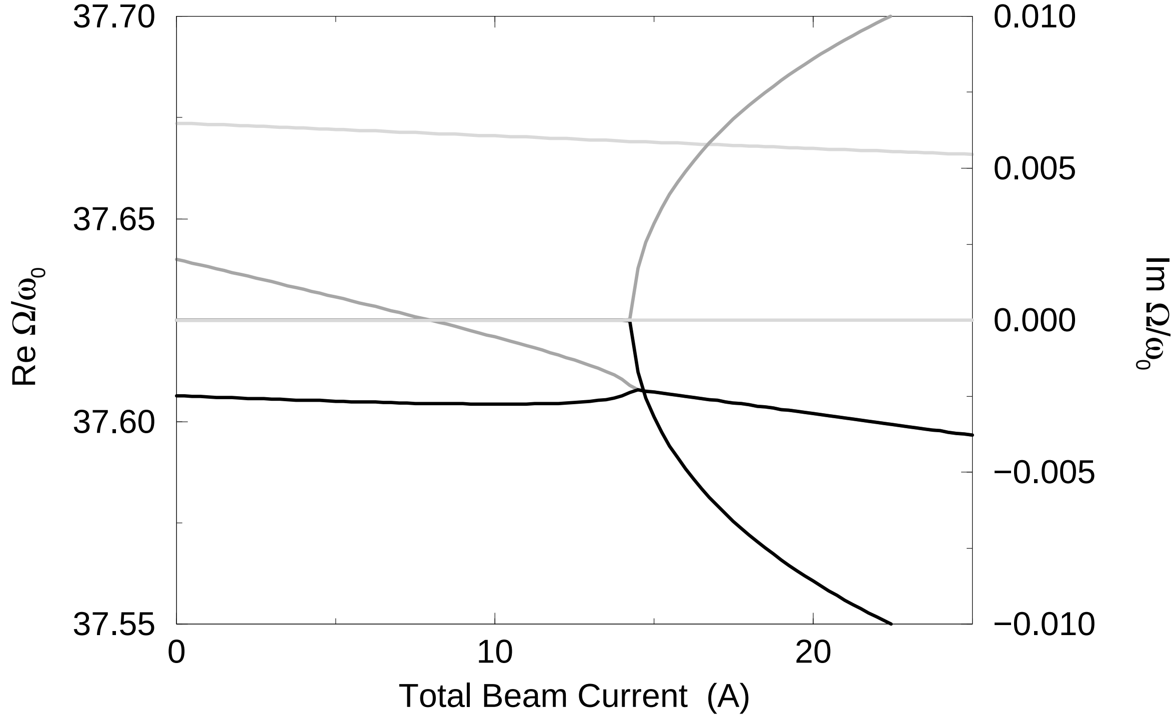

FIGURE 1: Single-bunch mode coupling, plotted versus total beam current for 1746 bunches. Real and imaginary parts of frequencies are shown on the same graph. Real frequencies shift with increasing current until two frequencies coincide. Those frequencies continue to be identical for higher currents. Imaginary parts are zero until the real parts coincide. The imaginary parts then have a nonzero value for higher currents. Note that real and imaginary parts that correspond to the same mode have the same line style.

mode coupling.

Multibunch mode coupling also has an effect at currents below where the mode frequencies coincide. This is because the finite growth rates of the multibunch modes effectively broaden the frequency of a multibunch mode, and thus coupling can occur at currents lower than the current where the real parts of the frequencies are equal. This effect can be seen in Fig. 8. The modes grow nonlinearly with current even at currents much lower than 10 A, which was the lowest current where the mode frequencies coincided (see Fig. 3). This effect can be seen more clearly when looking at the multibunch modes plotted for a fixed current. Fig. 9 shows these modes without coupling, whereas Fig. 10 shows these modes with coupling. These figures are plotted for 3.159 A, well below the 10 A where mode frequencies coincide. Without coupling, the $m = 1$ modes are nearly degenerate. When coupling is added, the growth rates of the $m = 1$ modes change significantly.

A feedback system in PEP-II is designed to damp the transverse rigid motion of the bunches. Such a system operates at relatively low frequencies. Thus, it fails to damp the $m = 1$ growth rates that result from multibunch mode coupling, as shown in Fig. 11.

The main problems that are seen from this analysis of PEP-II are $m = 1$ modes that have growth rates significantly above radiation damping, as can be seen in Fig. 10. These growth rates are primarily caused by cavity higher order modes at 1435 MHz and 1674 MHz. Some Landau damping [18, 19, 20, 21, 22, 23] is expected, but it is not expected to be large enough to damp these modes. Others are studying ways to damp these modes in the cavities.

5 Estimate of Mode Coupling Threshold

The sharp rise in growth rate for multibunch mode coupling occurs about when the real parts of the frequencies of the modes coincide. One can make a first approximation that the main change in the frequencies

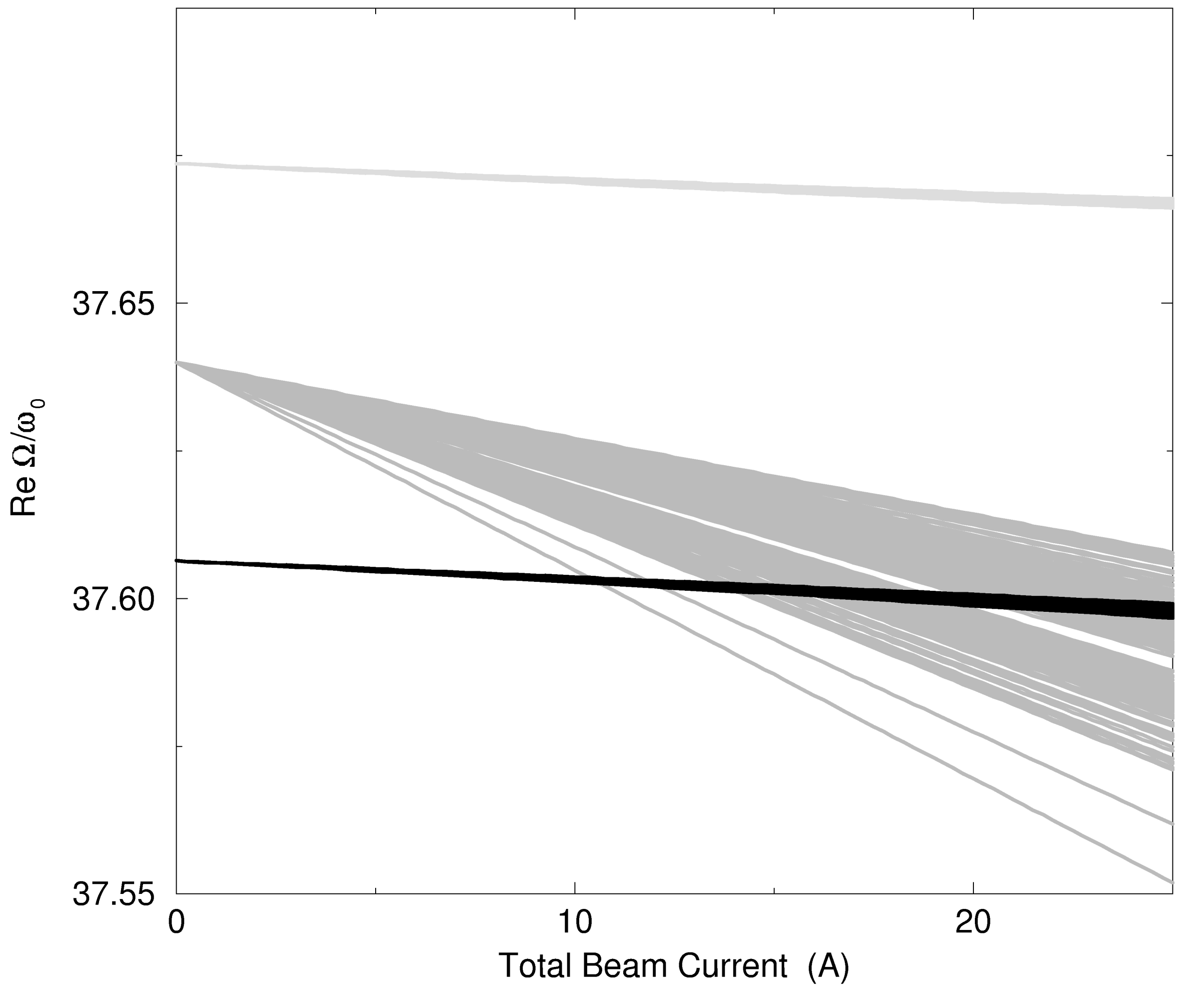

FIGURE 2: Multibunch mode frequencies, without coupling. Shown are three "fans" of 1746 lines each. Each line is the frequency of a single multibunch mode plotted versus current. The upper and lower fans contain the $m = 1$ modes, while the middle fan contains the $m = 0$ modes.

of the modes is in the shift in the $m = 0$ mode, ignoring coupling [10, 24]. Thus, once the frequency shift of the $m = 0$ mode is equal to $-\omega_s$, instability is expected.

Using equations (1–3) to compute the frequency shift of the $m = 0$ mode, the threshold current is approximately

$$I_{\text{th}} = -4\pi\nu_s\beta_0^2 \frac{E/e}{\beta_y Z_{\text{eff}}} \tag{8}$$

$$Z_{\text{eff}} = \sum_{\alpha} Im\left\{Z_\perp\big(\Omega + (p_0 + M\alpha)\omega_0\big)\right\} e^{-\sigma_\ell^2[\Omega+(p_0+M\alpha)\omega_0]^2/\beta_0^2c^2}. \tag{9}$$

The contribution to Z_{eff} can be separated into a piece due to broadband impedances, and a piece due to narrow-band impedances.

The piece due to broadband impedances is assumed to vary slowly, even over the scale of the bunch frequency $M\omega_0$. Thus, Z_{eff} can be approximated as an integral

$$Z_{\text{eff}}^{\text{BB}} \approx \frac{1}{M\omega_0} \int Im\left\{Z_\perp(\omega)\right\} e^{-\sigma_\ell^2\omega^2/\beta_0^2c^2} d\omega. \tag{10}$$

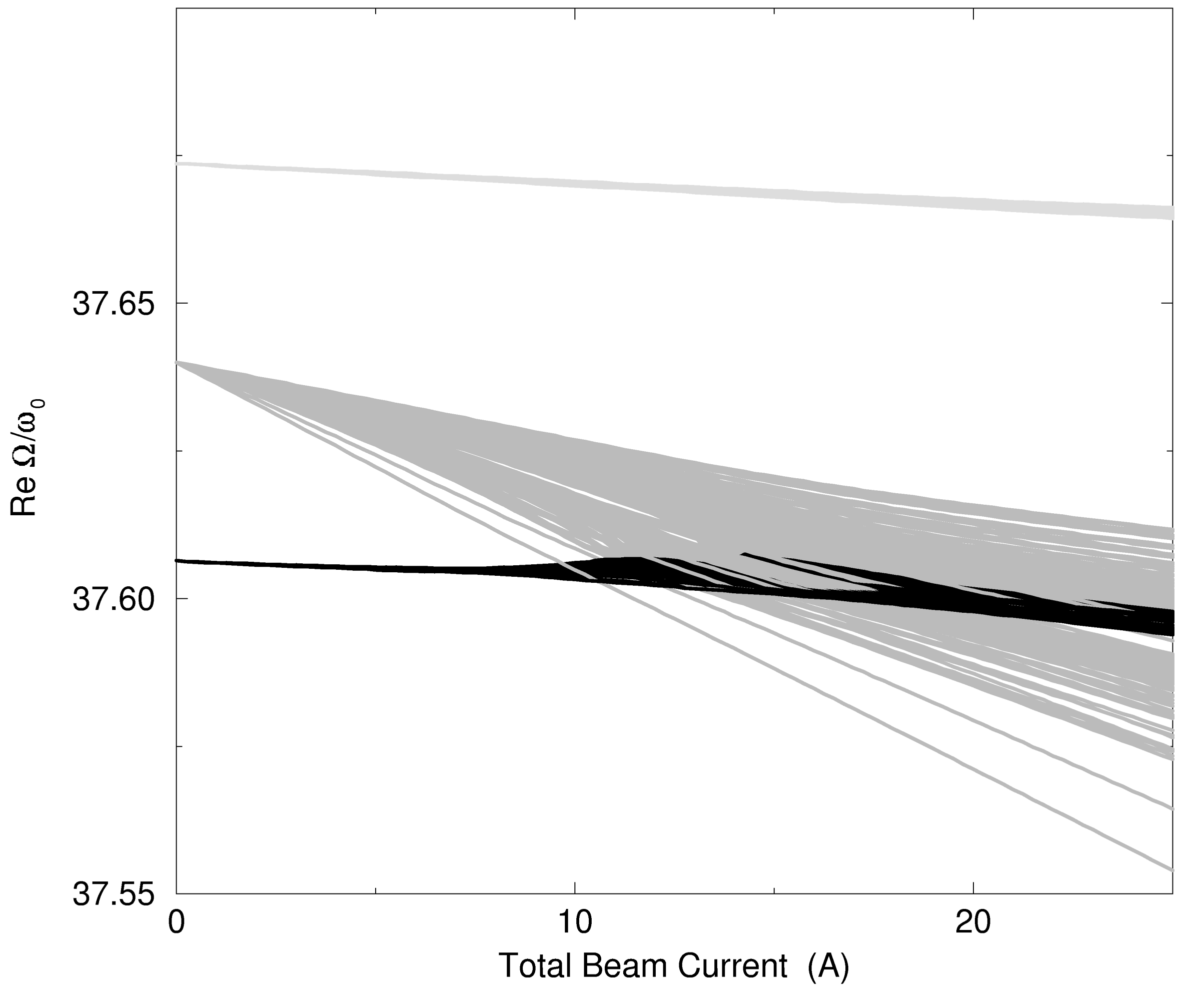

FIGURE 3: Multibunch mode frequencies, with coupling. Compare with Fig. 2.

Note that the $1/M$ dependence makes the threshold current per bunch independent of the number of bunches, as expected. If the broadband impedance is assumed to be constant over the bunch spectrum, this simplifies to

$$Z_{\text{eff}}^{\text{BB}} \approx \frac{L}{2\sqrt{\pi} M \sigma_\ell} Im\{Z_\perp\} \tag{11}$$

If the main contribution to the impedance is from a scaled inductance of 83.3 nH as in section 3.2 (ignoring the roll-off), Eqs. (8) and (11) predict a Z_{eff} of 2.84 MΩ/m, and therefore a mode coupling threshold of approximately 20.1 A. This result compares favorably to the actual threshold of 14.5 A, especially considering that many other sources of impedance have been ignored.

The contribution to Z_{eff} from narrow-band impedances can be computed by taking the peak of the narrow-band impedance. In most cases, it is only necessary to take a single term. However, if impedances are separated by a multiple of the bunch frequency, then they must be added together. The largest narrow-band impedance in PEP-II is the peak of the resistive wall, which is at 2.68 MΩ/m. Adding this to the Z_{eff} from the broadband impedance gives a mode coupling threshold of 10.33 A. This threshold agrees very well with where the mode coupling is beginning to have its strongest increase (see Figs. 3 and 7).

These estimates must be considered approximate, not only because other sources of frequency shift (from the coupling term, for instance) have been ignored, but also because mode coupling doesn't give a sharp threshold in the multibunch case; the effect occurs even at currents lower than where the frequencies coincide.

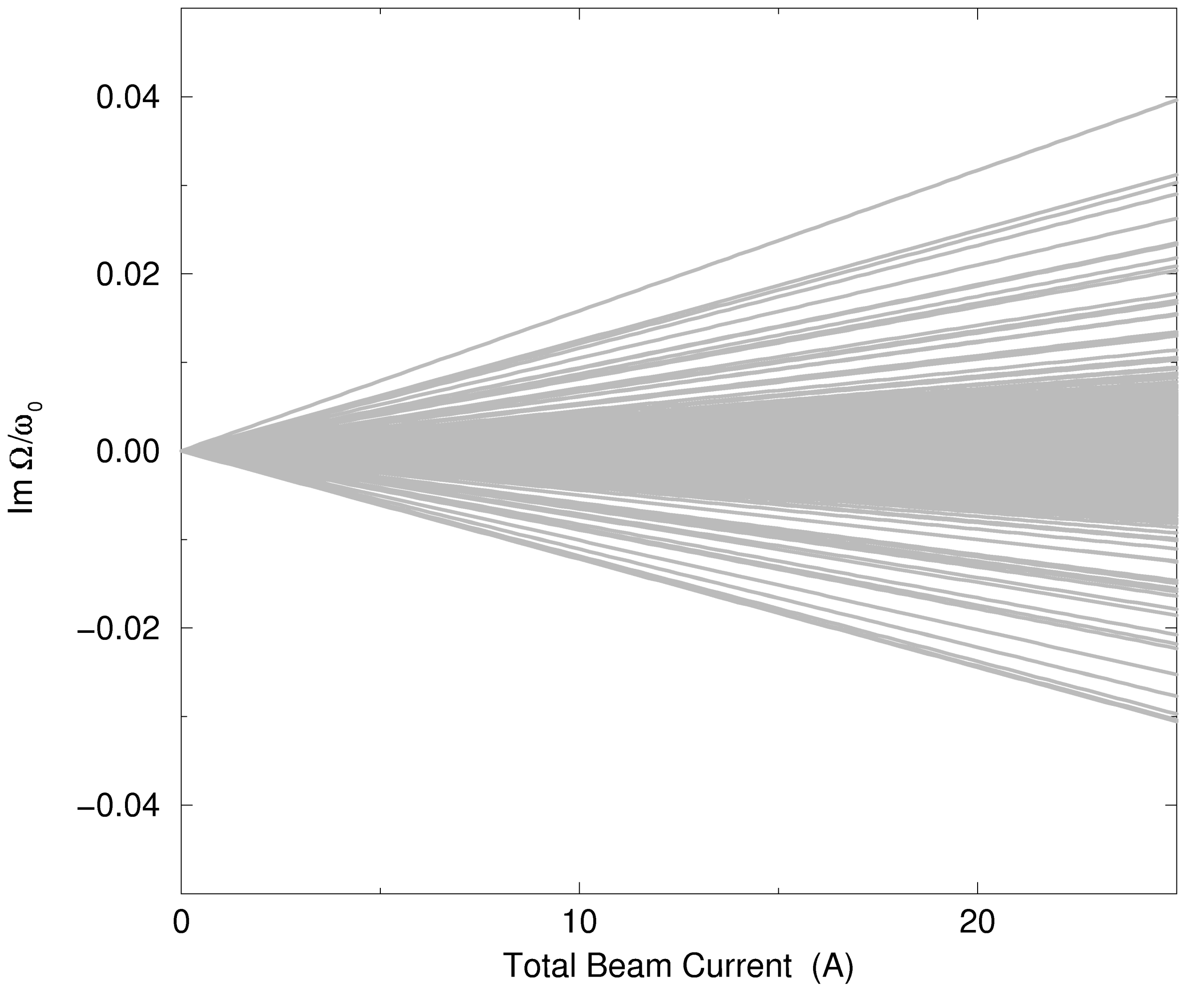

FIGURE 4: Multibunch $m = 0$ growth rates, no coupling. All 1746 modes are shown.

6 Conclusions

Multibunch mode coupling can cause significant increases in the growth rates of multibunch modes. The strongest effects are seen in $m = 1$ multibunch modes. The effect occurs at currents that are lower than the current where mode coupling would occur if only a single bunch were considered. The effect can be significant even well below the current where the frequencies of the multibunch modes coincide, although its strength increases rapidly at that current.

While this effect is fairly small in PEP-II, it is also clear that this machine is pushing the boundary of the importance of multibunch mode coupling.

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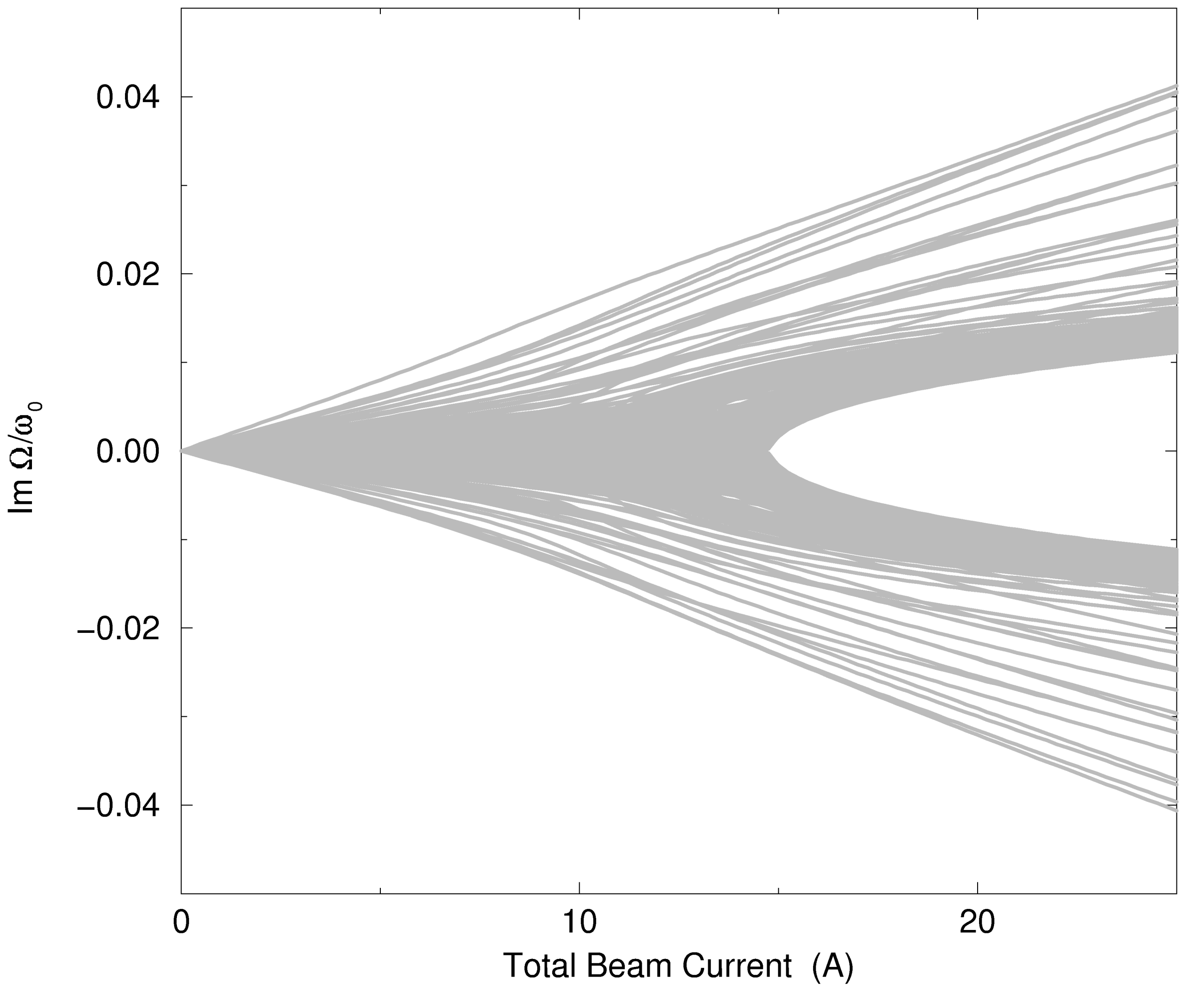

FIGURE 5: Multibunch $m = 0$ growth rates, with coupling.

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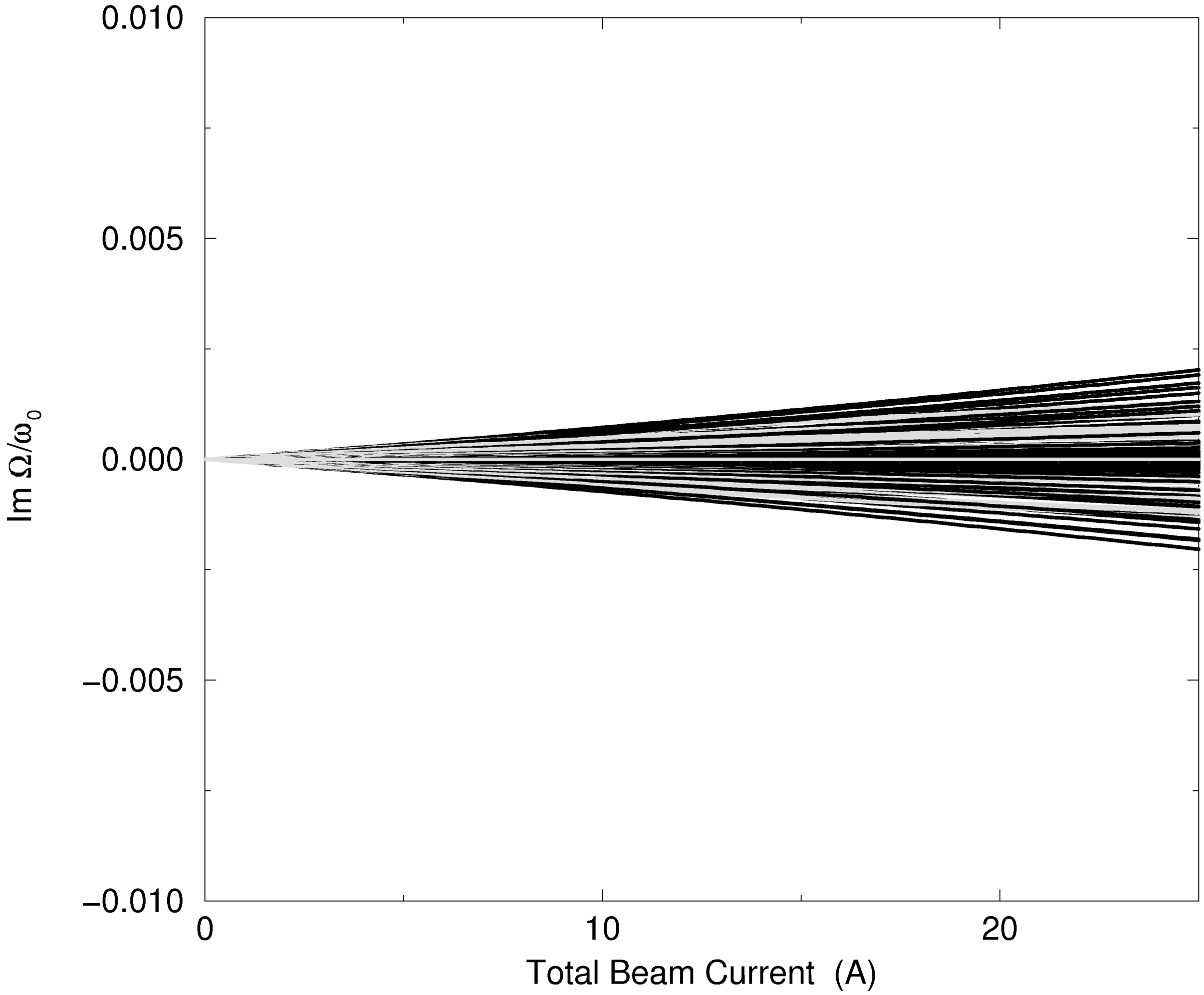

FIGURE 6: Multibunch $m = 1$ growth rates, no coupling. The 3492 mode lines correspond to two sets of 1746 $m = 1$ modes.

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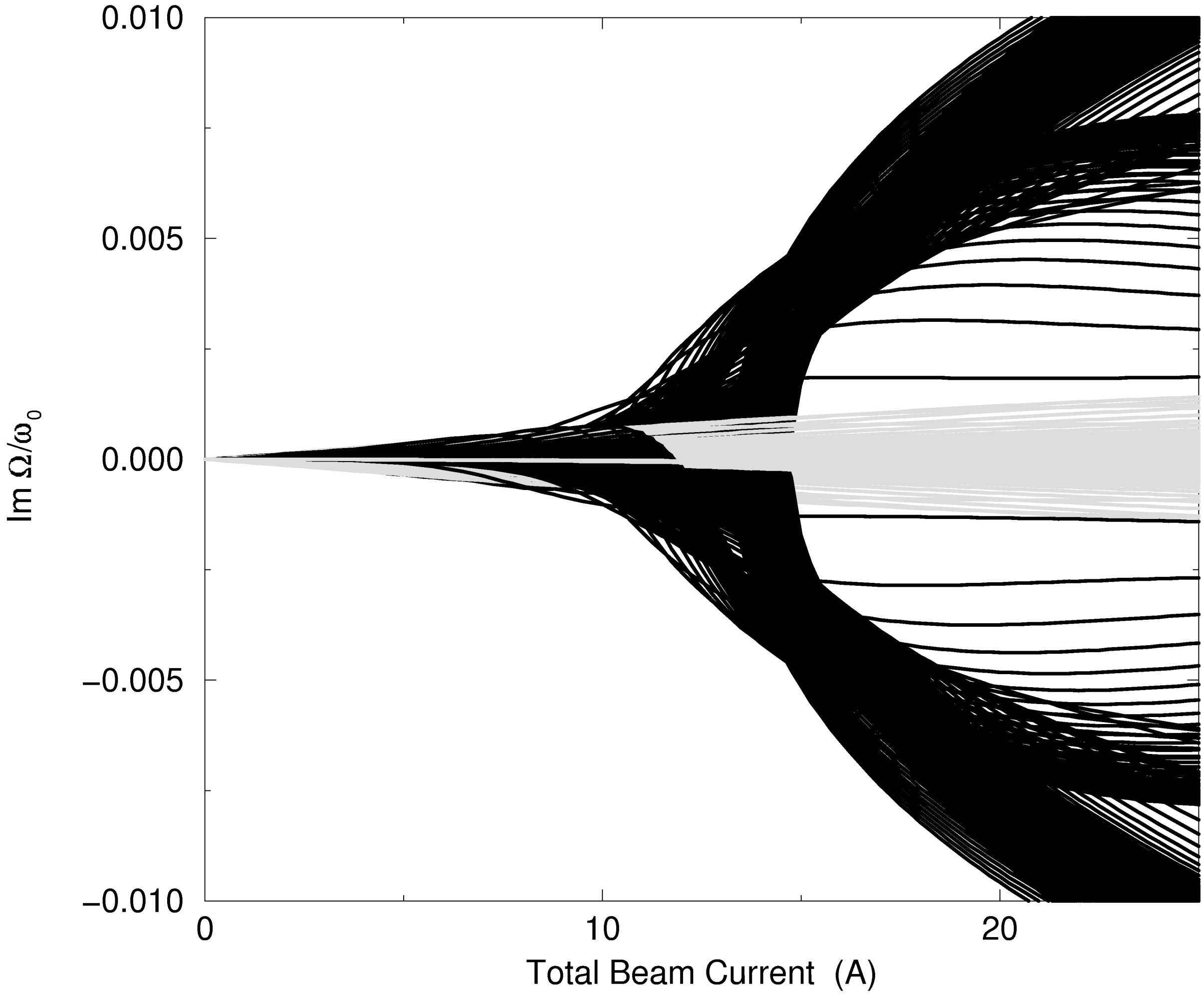

FIGURE 7: Multibunch $m = 1$ growth rates, with coupling. Lighter shaded lines are $\nu_\beta + \nu_s$ lines, darker lines are $\nu_\beta - \nu_s$ lines.

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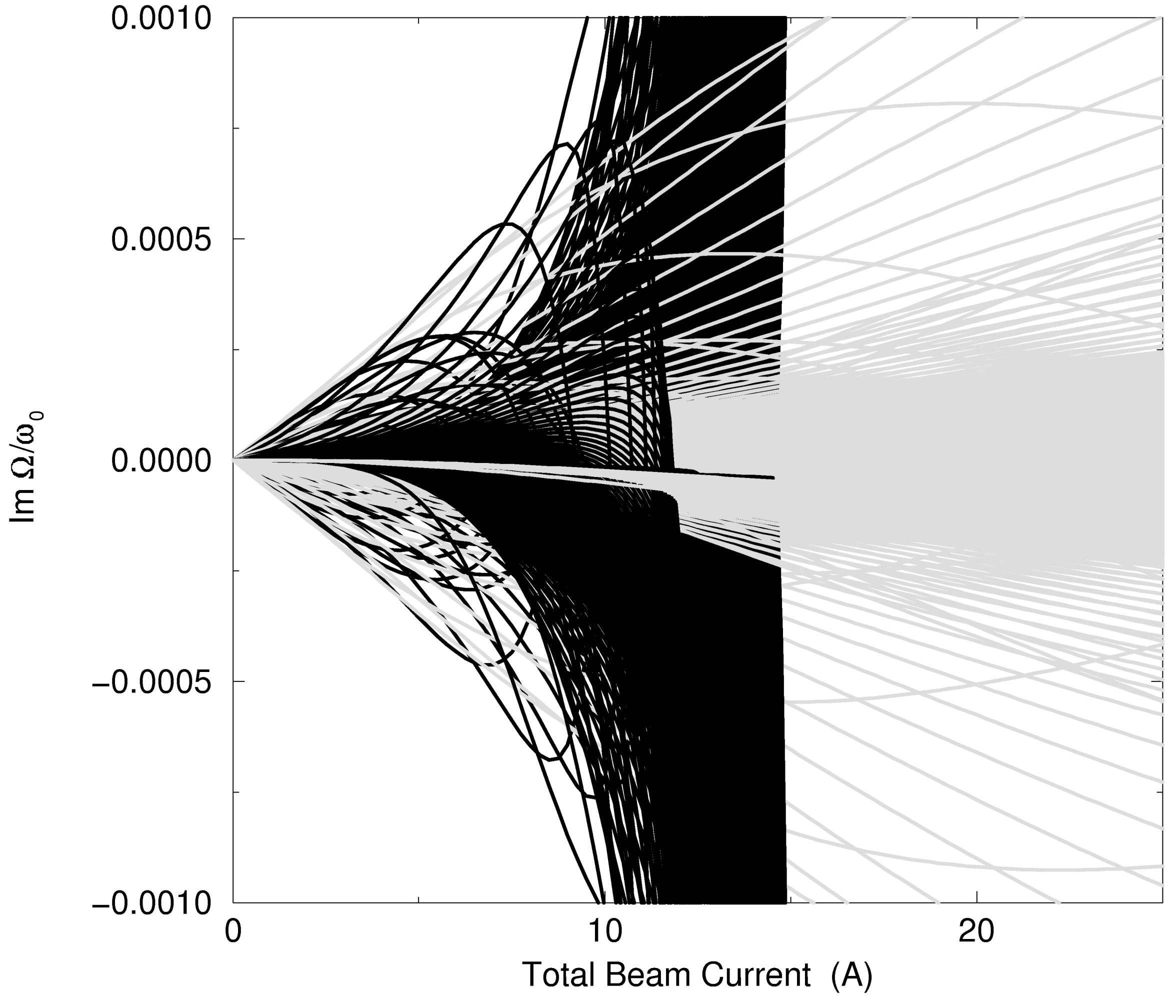

FIGURE 8: Multibunch $m = 1$ growth rates, with coupling. Vertical scale blown up a factor of 10 from Fig. 7.

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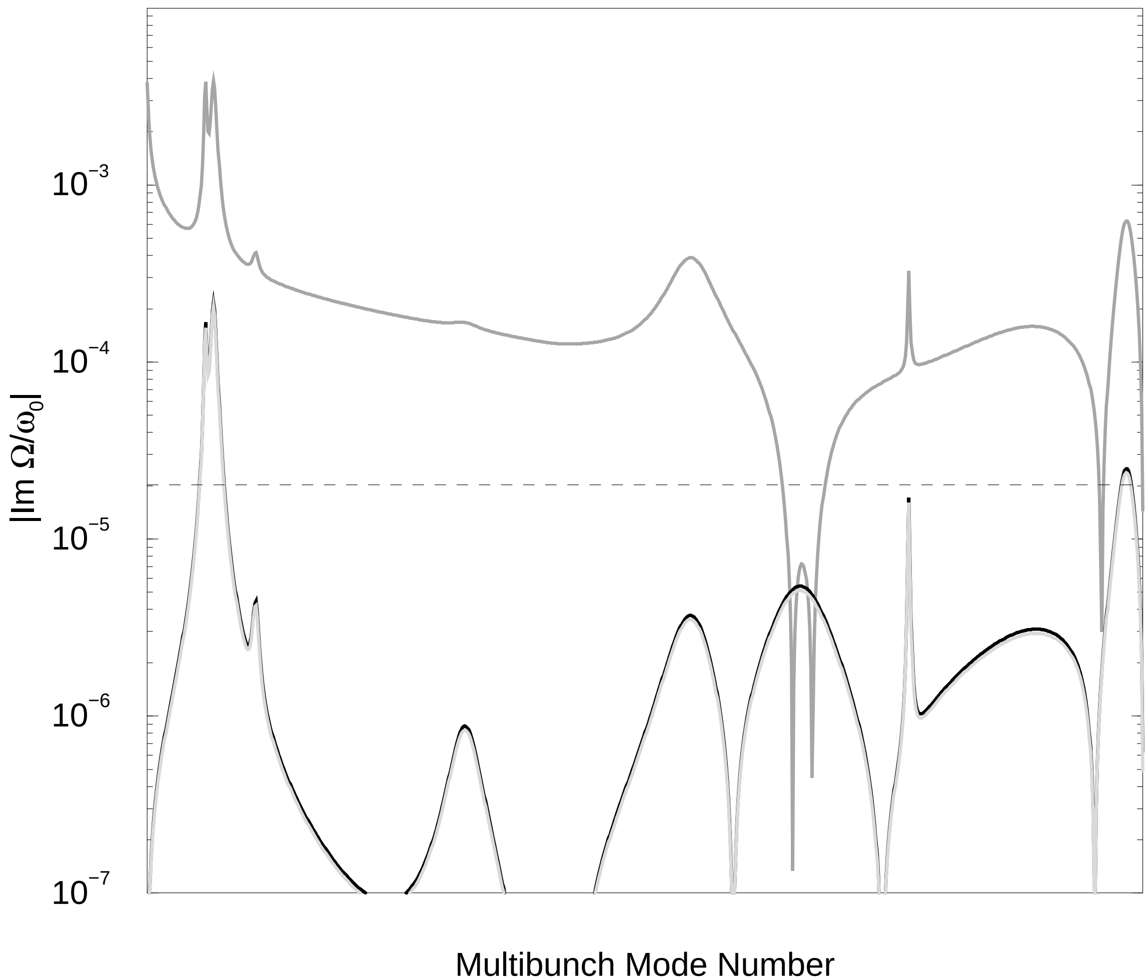

FIGURE 9: Multibunch modes at 3.159 A total beam current. Coupling is ignored. Note that the two $m = 1$ modes are nearly degenerate. The horizontal axis is the frequency offset of the mode. The lines are actually 873 points connected by lines.

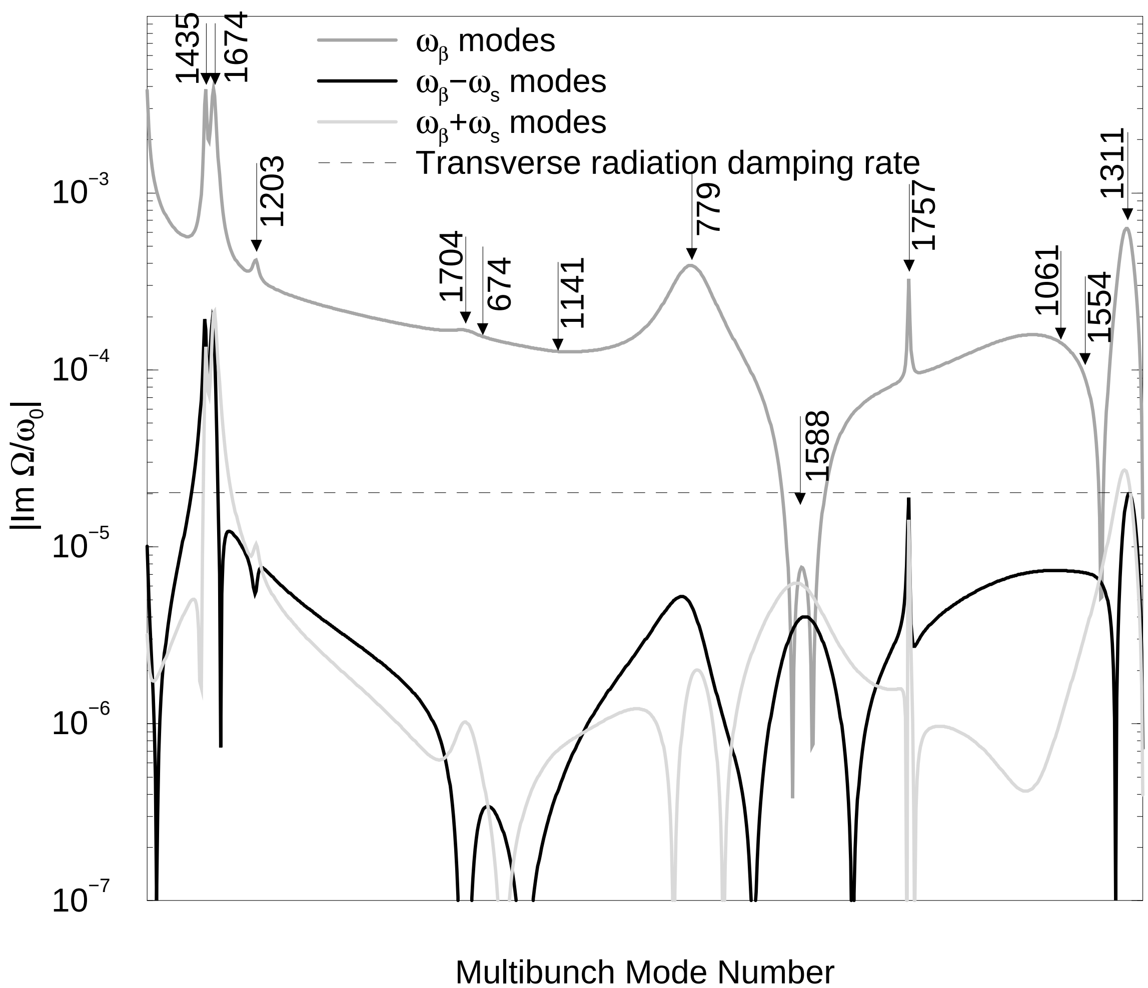

FIGURE 10: Multibunch modes at 3.159 A total beam current, with coupling. Arrows are numbered with the frequencies (in MHz) of the cavity higher order modes. The horizontal axis is the frequency offset of the mode. The large peak corresponding to the 1435 MHz cavity mode is enhanced nontrivially by multibunch mode coupling.

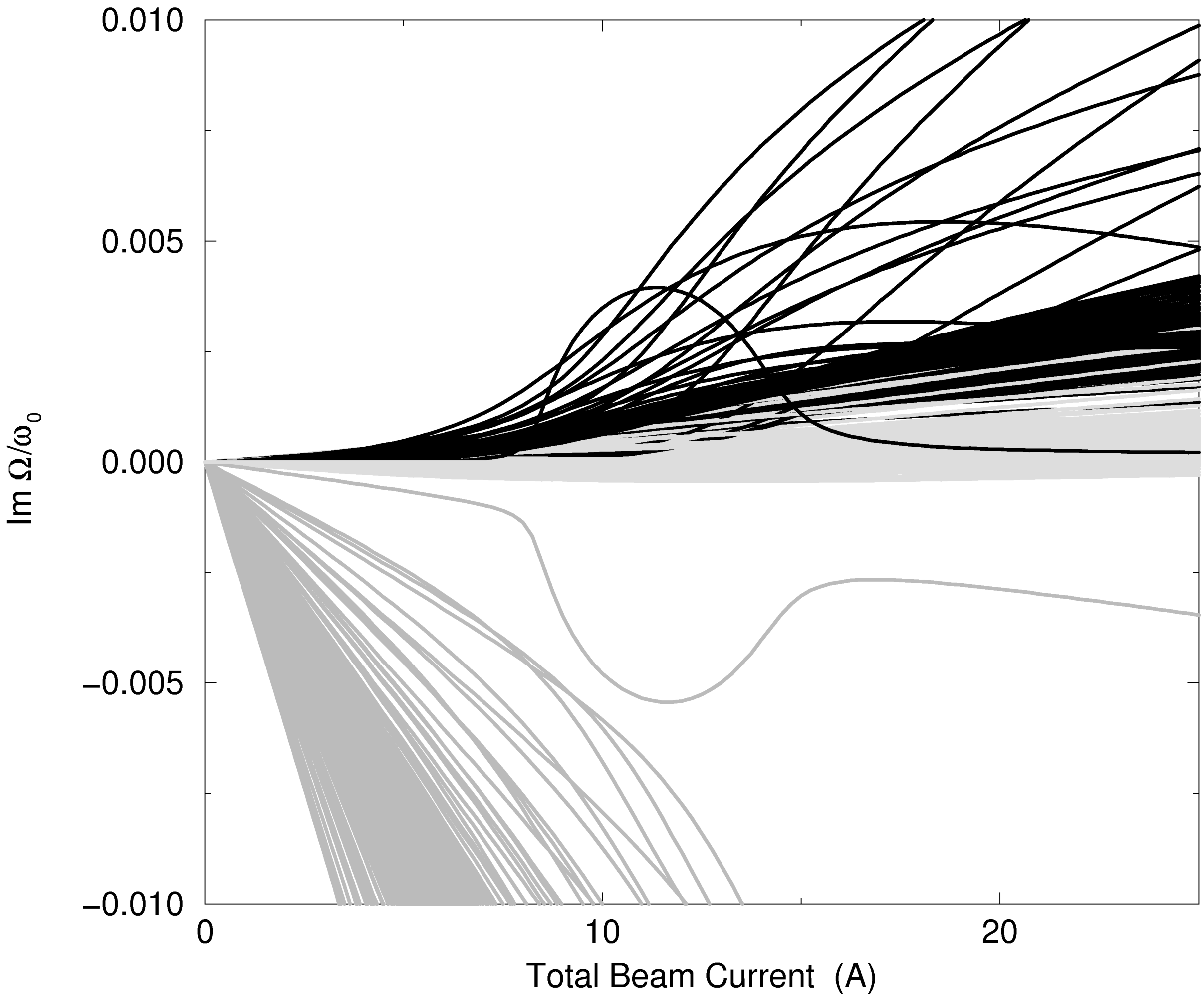

FIGURE 11: Multibunch growth rates, with feedback. The feedback is modelled as a Gaussian response about zero frequency with standard deviation of 125 MHz. Lower lines are $m = 0$ growth rates, upper lines are $m = 1$.